# Optimal Labor Income Taxation 

ECON 3003<br>Advanced Public Economics

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"The hardest thing in the worl to understand is the income tax."

- Albert Einstein


## GOALS OF NEXT TWO LECTURES

To prove Einstein wrong!

1) Understand the core optimal income tax model: linear and nonlinear taxes in the Saez (2001) framework $\square$

- Understand the equity-efficiency trade-off
- Revenue-maximizing tax rate (Laffer curve)
- Optimal linear tax rate formula
- Optimal top tax rate

2) Study the optimal design of transfer programs

- With only intensive margin responses
- Introduce extensive margin responses
- Tagging and in-kind programs


## TAXATION AND REDISTRIBUTION

Key question: By how much should government reduce inequality using taxes and transfers?

1) Governments use taxes to raise revenue and fund transfer programs which can reduce inequality in disposable income
2) Taxes (and transfers) create economic inefficiency if individuals are very responsive (work less, avoid/evade taxes)

Size of behavioural response limits the ability of government to redistribute with taxes/transfers

Let's study the standard optimal model to see why...

## KEY CONCEPTS FOR TAXES/TRANSFERS

Draw budget $(z, z-T(z))$ which integrates taxes and transfers

1) Transfer benefit with zero earnings - T(0) [sometimes called demogrant or lumpsum grant]
2) Marginal tax rate (or phasing-out rate) $T^{\prime}(z)$ : individual keeps $1-T^{\prime}(z)$ for an additional $\$ 1$ of earnings (matters for intensive labor supply response)
3) Participation tax rate $(\mathrm{PTR}) \tau_{p}=[T(z)-T(0)] / z$ : individual keeps fraction $1-\tau_{p}$ of earnings when moving from zero earnings to earnings $z$ (matters for extensive labor supply response):

$$
z-T(z)=-T(0)+z-[T(z)-T(0)]=-T(0)+z \cdot\left(1-\tau_{p}\right)
$$

4) Break-even earnings point $z^{*}$ : point at which $T\left(z^{*}\right)=0$



US Tax/Transfer System, single parent with 2 children, 2009


Source: Comput at ions made by Enmanuel Saez using tax and transfer system par amet er s


Sour ce: Pi ketty, Thomas, and Emmanuel Saez (2012)

## Profile of Current Means-tested Transfers

Traditional means-tested programs reduce incentives to work for low income workers

Refundable tax credits have significantly increased incentive to work for low income workers

However, refundable tax credits cannot benefit those with zero earnings
Trade-off: US chooses to reward work more than most European countries (such as France or the UK) but therefore provides smaller benefits to those with no earnings

## OPTIMAL INCOME TAXATION

> Goals

## Optimal Taxation: Case with No Behavioral Responses

- Utility $u(c)$ strictly increasing and concave on after-tax income $c$. Same $u(c)$ for everybody
- Income $z$ is fixed for each individual, $c=z-T(z)$ where $T(z)$ is tax/transfer on $z($ tax if $T(z)>0$, transfer if $T(z)<0)$
- $N$ individuals with fixed incomes $z_{1}<\ldots<z_{N}$
- Government maximizes Utilitarian objective: SWF $=\sum_{i=1}^{N} u\left(z_{i}-T\left(z_{i}\right)\right)$ subject to budget constraint $\sum_{i=1}^{N} T\left(z_{i}\right)=0$ (taxes need to fund transfers)


## Simple Model With No Behavioral Responses

Replace $T\left(z_{1}\right)=-\sum_{i=2}^{N} T\left(z_{i}\right)$ from budget constraint:

$$
S W F=u\left(z_{1}+\sum_{i=2}^{N} T\left(z_{i}\right)\right)+\sum_{i=2}^{N} u\left(z_{i}-T\left(z_{i}\right)\right)
$$

First order condition (FOC) in $T\left(z_{j}\right)$ for a given $j=2, . ., N$ :

$$
0=\frac{\partial S W F}{\partial T\left(z_{j}\right)}=u^{\prime}\left(z_{1}+\sum_{i=2}^{N} T\left(z_{i}\right)\right)-u^{\prime}\left(z_{j}-T\left(z_{j}\right)\right)=0 \Rightarrow
$$

$u^{\prime}\left(z_{j}-T\left(z_{j}\right)\right)=u^{\prime}\left(z_{1}-T\left(z_{1}\right)\right) \Rightarrow z_{j}-T\left(z_{j}\right)=$ constant for $j=1, . ., N$
Perfect equalization of after-tax income $=100 \%$ MTR and redistrib
Utilitarianism with decreasing marginal utility leads to perfect egalitarianism [Edgeworth, 1897]

## Simpler Derivation with just 2 individuals

$$
\max S W F=u\left(z_{1}-T\left(z_{1}\right)\right)+u\left(z_{2}-T\left(z_{2}\right)\right) \text { s.t. } T\left(z_{1}\right)+T\left(z_{2}\right)=0
$$

Replace $T\left(z_{1}\right)=-T\left(z_{2}\right)$ in SWF using budget constraint:

$$
S W F=u\left(z_{1}+T\left(z_{2}\right)\right)+u\left(z_{2}-T\left(z_{2}\right)\right)
$$

First order condition (FOC) in $T\left(z_{2}\right)$ :

$$
0=\frac{d S W F}{d T\left(z_{2}\right)}=u^{\prime}\left(z_{1}+T\left(z_{2}\right)\right)-u^{\prime}\left(z_{2}-T\left(z_{2}\right)\right)=0 \Rightarrow
$$

$u^{\prime}\left(z_{1}+T\left(z_{2}\right)\right)=u^{\prime}\left(z_{2}-T\left(z_{2}\right)\right) \Rightarrow u^{\prime}\left(z_{1}-T\left(z_{1}\right)\right)=u^{\prime}\left(z_{2}-T\left(z_{2}\right)\right)$
$\Rightarrow z_{1}-T\left(z_{1}\right)=z_{2}-T\left(z_{2}\right)$ constant across the 2 individuals
Perfect equalization of after-tax income $=100 \%$ marginal tax rate and redistribution [see graph]


## Optimal Tax/Transfer Systems



## ISSUES WITH SIMPLE MODEL

1) No behavioural responses: Obvious missing piece: $100 \%$
redistribution would destroy incentives to work and thus the assumption that $z$ is exogenous is unrealistic
$\Rightarrow$ Optimal income tax theory incorporates behavioural responses
2) Issue with Utilitarianism: Even absent behavioural responses, many people would object to $100 \%$ redistribution [perceived as confiscatory]
$\Rightarrow$ Citizens' views on fairness impose bounds on redistribution govt can do [political economy / public choice theory]

## EQUITY-EFFICIENCY TRADE-OFF

Taxes can be used to raise revenue for transfer programs which can reduce inequality in disposable income
$\Rightarrow$ Desirable if society feels that inequality is too large
Taxes (and transfers) reduce incentives to work
$\Rightarrow$ High tax rates create economic inefficiency if individuals respond to taxes

Size of behavioral response limits the ability of government to redistribute with taxes/transfers
$\Rightarrow$ Generates an equity-efficiency trade-off
Empirical tax literature estimates the size of behavioral responses

## Optimal Tax/Transfer Systems



## LABOR SUPPLY THEORY

Individual has utility over labor supply $I$ and consumption $c: u(c, I)$ increasing in $c$ and decreasing in $/$ [= increasing in leisure]

$$
\max _{c, l} u(c, I) \quad \text { subject to } \quad c=w \cdot l+R
$$

with $w=\bar{w} \cdot(1-\tau)$ the net-of-tax wage $(\bar{w}$ is before tax wage rate and $\tau$ is tax rate), and $R$ non-labor income

FOC $w \frac{\partial u}{\partial c}+\frac{\partial u}{\partial l}=0$ defines Marshallian labor supply $I=I(w, R)$
Uncompensated labor supply elasticity: $\quad \varepsilon^{u}=\frac{w}{l} \cdot \frac{\partial l}{\partial w}$
Income effects: $\quad \eta=w \frac{\partial I}{\partial R} \leq 0$ (if leisure is a normal good)





## Labor Supply Theory

Substitution effects: Hicksian labor supply: $I^{C}(w, u)$ minimizes cost needed to reach $u$ given slope $w \Rightarrow$

$$
\text { Compensated elasticity: } \quad \varepsilon^{c}=\frac{w}{l} \cdot \frac{\partial l^{c}}{\partial w}>0
$$

Slutsky equation: $\quad \frac{\partial I}{\partial w}=\frac{\partial I^{c}}{\partial w}+I \frac{\partial I}{\partial R} \Rightarrow \varepsilon^{u}=\varepsilon^{c}+\eta$
Marginal tax rate $\tau$ discourages work through substitution effects (working pays less at the margin)

Marginal tax rate $\tau$ encourages work through income effects (taxes make you poorer and hence in more need of income)

Net effect ambiguous (captured by sign of $\varepsilon^{u}$ )








## General nonlinear income tax

With no taxes: $c=z$ (consumption $=$ earnings)
With taxes $c=z-T(z)$ (consumption $=$ earnings - net taxes)
$T(z) \geq 0$ if individual pays taxes on net, $T(z) \leq 0$ if individual receives transfers on net
$T^{\prime}(z)>0$ reduces net wage rate and reduces labor supply through substitution effects
$T(z)>0$ reduces disposable income and increases labor supply through income effects
$T(z)<0$ increases disposable income and decreases labor supply through income effects

Transfer program such that $T(z)<0$ and $T^{\prime}(z)>0$ always discourages labor supply [see next graph when $z<z^{*}$ ]

## Effect of Taxes/Transfers on Labor Supply



## Effect of Taxes/Transfers on Labor Supply



## Effect of Taxes/Transfers on Labor Supply



## OPTIMAL LINEAR TAX RATE: LAFFER CURVE

$c=(1-\tau) \cdot z+R$ with $\tau$ linear tax rate and $R$ fixed universal transfer funded by taxes $R=\tau \cdot Z$ with $Z$ average earnings

Individual $i=1, . ., N$ chooses $I_{i}$ to $\max u^{i}\left((1-\tau) \cdot w_{i} I_{i}+R, l_{i}\right)$
Labor supply choices $l_{i}$ determine individual earnings $z_{i}=w_{i} l_{i} \Rightarrow$ Average earnings $Z=\sum_{i} z_{i} / N$ depends (positively) on net-of-tax rate $1-\tau$

Tax Revenue per person $R(\tau)=\tau \cdot Z(1-\tau)$ is inversely U-shaped with $\tau: R(\tau=0)=0$ (no taxes) and $R(\tau=1)=0$ (nobody works): called the Laffer Curve


## OPTIMAL LINEAR TAX RATE: LAFFER CURVE

Top of the Laffer Curve is at $\tau^{*}$ maximizing tax revenue:

$$
0=R^{\prime}\left(\tau^{*}\right)=Z-\tau^{*} \frac{d Z}{d(1-\tau)} \Rightarrow \frac{\tau^{*}}{1-\tau^{*}} \cdot \frac{1-\tau^{*}}{Z} \frac{d Z}{d(1-\tau)}=1
$$

Revenue maximizing tax rate: $\tau^{*}=\frac{1}{1+e}$ with $e=\frac{1-\tau}{Z} \frac{d Z}{d(1-\tau)}$
$e$ is the elasticity of average income $Z$ with respect to the net-of-tax rate $1-\tau$ [empirically estimable]

Inefficient to have $\tau>\tau^{*}$ because decreasing $\tau$ would make taxpayers better off (they pay less taxes) and would increase tax revenue for the government [and hence univ. transfer $R$ ]

If government is Rawlsian (i.e., maximizes welfare of the worst-off person with no earnings) then $\tau^{*}=1 /(1+e)$ is optimal to make transfer $R(\tau)$ as large as possible

## OPTIMAL LINEAR TAX RATE: FORMULA

Government chooses $\tau$ to maximize utilitarian social welfare

$$
S W F=\sum_{i} u^{i}\left((1-\tau) w_{i} l_{i}+\tau \cdot Z(1-\tau), l_{i}\right)
$$

taking into account that labor supply $l_{i}$ responds to taxation and hence that this affects the tax revenue per person $\tau \cdot Z(1-\tau)$ that is redistributed back as transfer to everybody

Government first order condition: (using the envelope theorem as $I_{i}$ maximizes $u^{i}$ ):

$$
0=\frac{d S W F}{d \tau}=\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot\left[-z_{i}+Z(.)-\tau \frac{d Z}{d(1-\tau)}\right]
$$

## OPTIMAL LINEAR TAX RATE: FORMULA

$$
\begin{gathered}
0=\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot\left[-z_{i}+Z(.)-\tau \frac{d Z}{d(1-\tau)}\right] \\
-\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot z_{i}+\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot Z(.)=\sum_{i} \frac{\partial u^{i}}{\partial c} \tau \frac{d Z}{d(1-\tau)}, \\
-\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot z_{i} \frac{1-\tau}{Z} \cdot \frac{1}{\sum_{i} \frac{\partial u^{\prime}}{\partial c}}+\sum_{i} \frac{\partial u^{i}}{\partial c} \cdot Z(.) \frac{1-\tau}{Z} \cdot \frac{1}{\sum_{i} \frac{\partial u}{\partial c}}=\tau \frac{1-\tau}{Z} \frac{d Z}{d(1-\tau)}, \\
-\bar{g} \cdot(1-\tau)+(1-\tau)=\tau \cdot e, \\
(1-\tau) \cdot(1-\bar{g})=\tau \cdot e, \\
\frac{1-\tau}{\tau}=\frac{e}{1-\bar{g}} \longrightarrow \frac{1}{\tau}=\frac{e}{1-\bar{g}}+1=\frac{1-\bar{g}+e}{1-\bar{g}} \\
\tau=\frac{1-\bar{g}}{1-\bar{g}+e}
\end{gathered}
$$

## OPTIMAL LINEAR TAX RATE: FORMULA

Hence, we have the following optimal linear income tax formula

$$
\tau=\frac{1-\bar{g}}{1-\bar{g}+e} \quad \text { with } \quad \bar{g}=\frac{\sum_{i} z_{i} \cdot \frac{\partial u^{i}}{\partial c}}{Z \cdot \sum_{i} \frac{\partial u^{i}}{\partial c}}
$$

$0 \leq \bar{g}<1$ as $\frac{\partial u^{i}}{\partial c}$ lower when income $z_{i}$ is high (marginal utility falls with consumption)
$\tau$ decreases with elasticity $e$ [efficiency] and with $\bar{g}$ [equity]
Formula captures the equity-efficiency trade-off
$\bar{g}$ is low and $\tau$ close to Laffer rate $\tau^{*}=1 /(1+e)$ when
(a) inequality is high
(b) marginal utility decreases fast with income

## OPTIMAL TOP INCOME TAX RATE

(Diamond and Saez JEP'11)

In practice, individual income tax is progressive with brackets with increasing marginal tax rates. What is the optimal top tax rate?

Consider constant MTR $\tau$ above fixed $z^{*}$ Goal: derive optimal $\tau$ In the UK, $\tau=45 \%$ and $z^{*}=£ 150,000(\simeq$ top $1 \%)$

Denote by $z$ average income of top bracket earners [depends on net-of-tax rate $1-\tau]$, with elasticity $e=[(1-\tau) / z] \cdot d z / d(1-\tau)$

Suppose the government wants to maximize tax revenue collected from top bracket taxpayers (marginal utility of consumption of top $1 \%$ earners is small)

## Optimal Top Income Tax Rate (Mirrlees ' 71 model)



Source: Diamond and Saez JEP'11

## Optimal Top Income Tax Rate (Mirrlees ' 71 model)



Source: Diamond and Saez JEP'11

## OPTIMAL TOP INCOME TAX RATE

Consider small $d \tau>0$ reform above $z^{*}$.

1) Mechanical increase in tax revenue:

$$
d M=\left[z-z^{*}\right] d \tau
$$

2) Behavioral response reduces tax revenue:

$$
\begin{gathered}
d B=\tau d z=-\tau \frac{d z}{d(1-\tau)} d \tau=-\frac{\tau}{1-\tau} \frac{1-\tau}{z} \frac{d z}{d(1-\tau)} \cdot z \cdot d \tau=-\frac{\tau}{1-\tau} \cdot e \cdot z \cdot d \tau \\
d M+d B=d \tau\left\{\left[z-z^{*}\right]-e \frac{\tau}{1-\tau} z\right\}
\end{gathered}
$$

Optimal $\tau$ such that $d M+d B=0$ :

$$
\Rightarrow \quad \frac{\tau}{1-\tau}=\frac{1}{e} \cdot \frac{z-z^{*}}{z} \Rightarrow \tau=\frac{1}{1+a \cdot e} \quad \text { with } \quad a=\frac{z}{z-z^{*}}
$$

## OPTIMAL TOP INCOME TAX RATE

Optimal top tax rate: $\tau=\frac{1}{1+a \cdot e} \quad$ with $\quad a=\frac{z}{z-z^{*}}$
Optimal $\tau$ decreases with e [efficiency]
Optimal $\tau$ decreases with a [thinness of top tail]
Empirically $a \in(1.5,3)$. US has $a \simeq 1.5$, UK has $a \simeq 1.67$, Denmark has $a \simeq 3$. Easy to estimate using distributional data [in the US, mean income above $z^{*}=\$ 0.5 \mathrm{~m}$ is about $\$ 1.5 \mathrm{~m}$ ]

Empirically $e$ is harder to estimate [controversial]
Example: If $e=0.25$ then $\tau=1 /(1+1.5 \cdot 0.25)=1 / 1.375=73 \%$

## OPTIMAL TOP INCOME TAX RATE Interpretation

(1) The more elastic rich people are (high e), the lower should optimal $\tau$ be (because of efficiency loss)
(2) Top rate depends negatively on the thinness of the top tail distribution. The higher a, the thinner is the tail. Intuitively, if the distrib is thin then $\uparrow$ top rate for high-income earners will raise little extra tax revenue $\Rightarrow$ a lower tax rate for the upper bracket is optimal

In fact, in the extreme case where there is only one person in the top bracket (the upper threshold is so high that it only includes the richest person), then $z$ is close to $z^{*}$, so $a \rightarrow \infty$ and $\tau \rightarrow 0$ (no tax for the richest person!)

But this is highly unrealistic as, empirically, there are usually more people in the upper bracket, which gives very stable values for a (e.g., 1.5 in the US, 3 in Denmark)

## REAL VS. TAX AVOIDANCE RESPONSES

Behavioral response to income tax comes not only from reduced labor supply but from tax avoidance or tax evasion

Tax avoidance: legal means to reduce tax liability (exploiting tax loopholes). E.g., untaxed fringe benefits.

Tax evasion: illegal under-reporting of income
Labor supply vs tax avoidance/evasion distinction matters because:

1) If people work less when tax rates increase, there is not much the government can do about it
2) If people avoid/evade more when tax rates increase, then the govt can reduce tax avoidance/evasion opportunities [close tax loopholes, broaden the tax base, increase tax enforcement, etc.]

## REAL VS. AVOIDANCE RESPONSES

Key policy question: Is it possible to eliminate avoidance responses using base broadening, etc.? or would new avoidance schemes keep popping up?
a) Some forms of tax avoidance are due to poorly designed tax codes (preferential treatment for some income forms or some deductions)
b) Some forms of tax avoidance/evasion can only be addressed with international cooperation (off-shore tax evasion in tax havens)
c) Some forms of tax avoidance/evasion are due to technological limitations of tax collection (impossible to tax informal cash businesses)

## EXTENSIONS AND LIMITATIONS

1) Model includes only intensive earnings response. Extensive earnings responses [entrepreneurship decisions, migration decisions] $\Rightarrow$ Formulas can be modified
2) Model does not include fiscal externalities: part of the response to $d \tau$ comes from income shifting which affects other taxes $\Rightarrow$ Formulas can be modified
3) Model does not include classical externalities: (a) charitable contributions, (b) positive spillovers (trickle down) [top earners underpaid], (c) negative spillovers [top earners overpaid]

Classical general equilibrium effects on prices are NOT externalities and do not affect formulas [Diamond-Mirrlees AER '71, Saez JpubE '04]

## REFERENCES

Jonathan Gruber, Public Finance and Public Policy, Fifth Edition, 2016 Worth Publishers, Chapter 20 and Chapter 21

Diamond, P. and E. Saez "From Basic Research to Policy Recommendations: The Case for a Progressive Tax", Journal of Economic Perspectives, 25.4, (2011): 165-190. (web)

Piketty, Thomas and Emmanuel Saez "Optimal Labor Income Taxation," Handbook of Public Economics, Volume 5, Amsterdam: Elsevier-North Holland, 2013. (web)

Saez, E. "Using Elasticities to Derive Optimal Income Tax Rates", Review of Economics Studies, Vol. 68, 2001, 205-229. (web)

Saez, Emmanuel. "Optimal income transfer programs: intensive versus extensive labor supply responses." The Quarterly Journal of Economics 117.3 (2002): 1039-1073 (web)

