Public Economics (ECON 131)
Section #10: Social Security and Insurance

April 10 and 12, 2017

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1 Key Concepts

- **Social insurance programs** are government interventions in the provision of insurance against adverse events.
- What is **insurance**? Why is it valuable to individuals?
- What is the **expected utility model**? How do you write up an agent's expected utility function?
- What is the **actuarially fair premium**?
- What is **asymmetric information**? What are the implications of asymmetric information in insurance markets?
- What is the difference between **adverse selection** and **moral hazard**? How do they relate to government interventions through social insurance?

2 Practice Problems

2.1 Adverse Selection

Do health insurance and car insurance markets both suffer from adverse selection? Are there reasons why the government should intervene in the health insurance market and not in the car market?

Solution:

- Yes, like most insurance markets, health and car suffer from adverse selection. Poor car drivers and sick people would want insurance the most.
- We have seen in lecture that in addition to dealing with adverse selection the government might want to intervene to
  (1) Redistribute based on negative health condition as it is seen out of the control of a person and fair
  (2) To internalize externalities (think vaccines)
  (3) To prevent individual failures which are more likely in a more complex market like health
  (4) To lower administrative costs that are huge in private health markets

2.2 Moral Hazard

Unemployment insurance in the US typically pays the unemployed 50% of their previous wage for about 6 months. During the recession of 2009 it was extended temporarily to a year. Discuss why unemployment insurance is far from 100% and why the government decided to extend UI during the recession.

Solution:

- 100% UI benefits would provide no incentives for job search until the end of their benefit period. This is a form of moral hazard, since in the absence of UI individuals would look fast for a new job and therefore get off the benefits.
- During the recession there were so few jobs available that not finding a job was hardly a sign of moral hazard.

2.3 Disability Insurance

Consider an economy where there are three types of people who want to buy disability insurance. Each type has the same health-based risks. They each have a 2 percent chance of being incapacitated due to health risks which are uncorrelated with their risk taking behavior. But the people differ in their hobbies and work. The low-risk types walk to work and have very low risk hobbies. Their outside risk of being incapacitated is 3 percent. The medium-risk types drive to work and actively play soccer on the weekends. Therefore, their non-health risk of being incapacitated is 8 percent. The third type has high risk. They work as firefighters and skydive on weekends. Therefore, their outside risk of being incapacitated is 58 percent. There are equal numbers of each type. Long-term care insurance provides income if they are incapacitated for the rest of their life (there are no additional costs). Individuals have the following utility function over consumption (or income):

$$u(c) = \log(c)$$

Individuals earn \$500 if healthy, but only \$10 if incapacitated. They make the decision to purchase insurance before the event occurs.

(a) *If the insurance company can differentiate the types, what is the socially optimal level of insurance for each type. [Note: no math is required for this question.]*

Solution:

All agents have concave utility functions, that is, the first derivative of the utility function is positive and the second derivative is negative which means they have diminishing marginal utility and are therefore risk averse. Risk averse agents fully insure when offered actuarially fair insurance, therefore all three types will choose to purchase insurance policies.

An insurance contract is actuarially fair if the insurance premium paid is set equal to the insurer's expected payout.

(b) *What is the actuarially fair price for the insurance for each group? What is the expected utility of each group given that price if they buy insurance? What is the expected utility if they do not buy insurance? What prices are each groups willing to pay for the insurance? Assume insurers are perfectly competitive.*

Solution:

First off, we know that the individual wants to purchase a policy with premium (or price) p and payout b to maximize their expected utility. Let q be the probability of an individual being incapacitated due to health risks, w the income of the individual when healthy and $w - d$ the income of the individual when incapacitated. Then, the expected utility of the individual (that the individual will maximize) can be written as:

$$\mathrm{E}[U] = (1 - q)\log(w - p) + q\log\left(w - d - p + b\right)$$

We know that $w = \$500$ and the loss of income due to disability is $d = \$490$, thus $w - d = \$10$.

In addition, if insurers are perfectly competitive, then firm profits equal $p - q * b = 0 \Rightarrow b = p/q$.

The individual's maximization problem is then:

$$\max_p \mathrm{E}[U] = (1-q)\log(500-p) + q\log\left(10-p+\frac{p}{q}\right)$$

FOC:

$$\frac{\partial \mathrm{E}[U]}{\partial p} = 0 = \frac{-(1-q)}{(500-p)} + \frac{q\left(-1+\frac{1}{q}\right)}{10-p+\frac{p}{q}}$$

$$\frac{1-q}{500-p} = \frac{1-q}{10-p+\frac{p}{q}}$$

$$490 = \frac{p}{q} = d$$

The actuarially fair optimal premium (and implicit benefit level) is given by: $p = d * \Pr(\text{disability}) = 490 * q$ and $b = d = 490$.

Therefore, the actuarially fair price for insurance for

- the low-risk types is $p = 490 * (0.05) = \$24.5$
- the medium-risk types is: $p = 490 * (0.10) = \$49$
- the high-risk types is: $p = 490 * (0.60) = \$294$

Expected utility of each group if they buy (full) insurance at price p is:

$$\mathrm{E}[U] = (1-q)\log(500-p) + q\log\left(10-p+\frac{p}{q}\right) = \log(500-p)$$

Therefore, the expected utility with insurance for

- the low-risk types is $\mathrm{E}(U) = \log(500-24.5) = 6.16$
- the medium-risk types is: $\mathrm{E}(U) = \log(500-49) = 6.11$
- the high-risk types is: $\mathrm{E}(U) = \log(500-294) = 5.33$

Expected utility of each group if they do not buy insurance is:

$$\mathrm{E}[U] = (1-q)\log(500) + q\log(10)$$

Therefore, the expected utility without insurance for

- the low-risk types is $\mathrm{E}[U] = (0.95)\log(500) + (0.05)\log(10) = 6.02$
- the medium-risk types is: $\mathrm{E}[U] = (0.90)\log(500) + (0.10)\log(10) = 5.82$
- the high-risk types is: $\mathrm{E}(U) = (0.40)\log(500) + (0.60)\log(10) = 3.87$

Finally, to be willing to pay for insurance, the expected utility from life with the insurance must be better than the expected utility from life without the insurance. To simplify, we will assume that individuals purchase enough insurance to fully insure themselves: $U_{ins} = \log(500 - p)$.

$$\begin{aligned}
\log(500 - p) &\geq (1 - q)\log(500) + q\log(10) \\
\log(500 - p) &\geq \log\left(500^{1-q} * 10^{q}\right) \\
500 - p &\geq 500^{1-q} * 10^{q} \\
p &\leq 500 - 500^{1-q} * 10^{q}
\end{aligned}$$

Therefore, the maximum price for

- the low-risk types is $p \leq 500 - 500^{0.95} * 10^{0.05} = \$88.83$
- the medium-risk types is: $p \leq 500 - 500^{0.90} * 10^{0.10} = \$161.88$
- the high-risk types is: $p \leq 500 - 500^{0.40} * 10^{0.60} = \$452.18$

(c) *Now assume that the insurance company is not able to differentiate between the three types. Therefore, it offers a policy that fully insures individuals at the same price to all three types. Assuming all three types buy this insurance policy, what is the price for insurance?*

Solution:

Assuming (1) firms make zero profits and (2) equal shares of each type in the population, the actuarially fair price is:

$$p = d * \mathrm{E}[\Pr(\text{disability})] = 490 * \frac{0.05 + 0.10 + 0.60}{3} = \$122.5$$

(d) *Prove that with the price you found in part (c), the low-risk type would not be willing to buy the full insurance policy. [Note: use your results from part (b).]*

Solution:

As you can see from part (c), the price of \$122.50 is greater than the maximum price that the low-risk type is willing to pay \$88.83. Therefore, the low-risk type will not buy the policy.

(e) *Since the low-risk type does not buy the policy, the insurance company cannot offer the policy at the price from part (c). What is the new price for this policy if the risk averse drop out of the market? Who will buy at this price? [Note: show mathematically.]*

Solution:

Without the low-risk types, the actuarially fair price is now:

$$p = d * \mathrm{E}[\Pr(\text{disability})] = 490 * \frac{0.10 + 0.60}{2} = \$171.5$$

Now the medium-risk types will refuse to buy the policy, since this is higher than their maximum (\$161.88). Therefore, only the high-risk types will buy (at their actuarially fair price of \$294) and the market has unraveled.

(f) *You have demonstrated an example of a market unraveling. Explain the intuition for why it happened.*

Solution:

- This market unraveling is called adverse selection.
- It is the result of asymmetric information between the insurance companies and those it may insure.
- The insurance company cannot distinguish between risk preferences, so it must offer the same policy to everyone, but in order to earn zero profit they must charge a premium equal to the population risk times the coverage amount.
- Since the risk adverse know who they are, they will not insure in order to maximize utility.
- This causes the risk of accident/disability among the insured population to increase, causing fewer people to insure, and so on until no insurance contract is offered at all even though everybody wants full actuarially fair insurance.

(g) *How would results change if the individuals' income is zero when incapacitated?*

Solution:

Given that at zero income the log utility is negative infinity, people would never choose not to be insured. We therefore obtain again a pooling equilibrium and we might not need government intervention. [Remember, a pooling equilibrium is such that insurance companies offer a contract based on average risk.]